Consider the graph of the polar equation $r = 8 - 5\sin\theta$.

SCORE: _____ / 6 PTS

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[a] Fill in the blanks.

$1 < \left|\frac{8}{-5}\right| = \frac{8}{5} < 2$
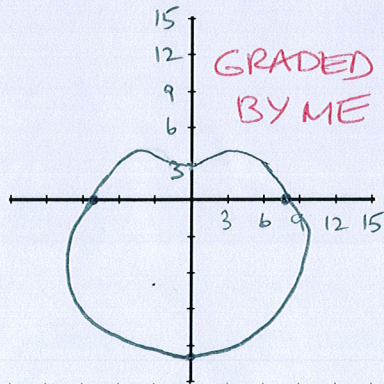
MUST HAVE "DIMPLE" TO EARN ANY POINTS

[i] The shape of the graph is a/an __LIMACON WITH DIMPLE__. ①

[ii] The graph __DOES NOT__ pass through the pole.
does / does not

[iii] Find the **rectangular coordinates** of the

x – intercept(s) ① $(\pm 8, 0)$

y – intercept(s) $(0, 3) (0, -13)$

$\frac{\theta}{0}$ | $\frac{r}{8}$
$\frac{\pi}{2}$ | 3
π | 8
$\frac{3\pi}{2}$ | 13

[b] Sketch the graph on the grid provided above. **You must provide a scale for the axes & plot all points from part [a][iii] above.**

Fill in the blanks.

SCORE: _____ / 4 PTS

[a] The polar co-ordinates $(-11, -\frac{8\pi}{5})$ correspond to the same point as the polar co-ordinates $(-11, \boxed{\frac{2\pi}{5}})$ and $(11, \boxed{\frac{7\pi}{5}})$.

NOTE: Both your answers for this question must be positive.

$-\frac{8\pi}{5} + 2\pi$ $\quad$ $\frac{2\pi}{5} + \pi$

[b] The point with rectangular co-ordinates $(-3\sqrt{2}, -\sqrt{6})$ has polar co-ordinates $(\boxed{2\sqrt{6}}, \boxed{\frac{7\pi}{6}})$.

NOTE: Both parts of your answer for this question must be positive.

$r^2 = (-3\sqrt{2})^2 + (-\sqrt{6})^2 = 18 + 6$
$= 24$

$\theta = \pi + \tan^{-1}\frac{-\sqrt{6}}{-3\sqrt{2}}$
$= \pi + \tan^{-1}\frac{\sqrt{3}}{3} = \pi + \frac{\pi}{6}$

[c] The point with polar co-ordinates $(8, -\frac{2\pi}{3})$ has rectangular co-ordinates $(\boxed{-4}, \boxed{-4\sqrt{3}})$.

$x = 8\cos\left(-\frac{2\pi}{3}\right) = 8 \cdot -\frac{1}{2}$ $\qquad$ $y = 8\sin\left(-\frac{2\pi}{3}\right) = 8 \cdot \frac{-\sqrt{3}}{2}$

Convert the polar equation $r = 2 - 7 \sin 2\theta$ to rectangular, and simplify as shown in the website handout.   SCORE: _____ / 5 PTS

NOTE: Your final answer should NOT have fractions, fractional exponents nor radicals.

$$r = 2 - 7(2\sin\theta\cos\theta)$$

$$r = 2 - 14 \frac{y}{r} \frac{x}{r}$$

$$r^3 = 2r^2 - 14xy$$

$$(\sqrt{x^2+y^2})^3 = 2(x^2+y^2) - 14xy$$

$$= 2x^2 - 14xy + 2y^2$$

$$(x^2+y^2)^3 = (2x^2 - 14xy + 2y^2)^2$$

① POINT EACH

Consider the graph of the polar equation $r = 4 - 8\cos 4\theta$. SCORE: _____ / 15 PTS

NOTE: $(-r, \theta)$ and $(-r, \pi-\theta)$ tests do NOT show that the graph is symmetric

POLE   POLAR AXIS

[a] **Using the information above, and the tests and shortcuts shown in lecture**, test if the graph is symmetric over the pole, the polar axis, and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

POLE: $r = 4 - 8\cos 4(\pi + \theta)$ ①
$r = 4 - 8\cos(4\pi + 4\theta)$
$r = 4 - 8[\cos 4\pi \cos 4\theta - \sin 4\pi \sin 4\theta]$
① $r = 4 - 8\cos 4\theta$, SYM

POLAR: $r = 4 - 8\cos 4(-\theta)$ ①
AXIS ① $r = 4 - 8\cos 4\theta$, SYM

AUTOMATICALLY SYM OVER $\theta = \frac{\pi}{2}$

Type of symmetry	Conclusion
Over the polar axis	SYMMETRIC
Over $\theta = \frac{\pi}{2}$	SYMMETRIC
Over the pole	SYMMETRIC

EXACTLY
① POINT IF 2 CORRECT
② POINTS IF ALL CORRECT

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot first (before using reflections to draw the rest of the graph) ?

$[0, \frac{\pi}{2}]$ ①

[c] Find the angles **algebraically** in the minimum interval in part [b] at which the graph goes through the pole.
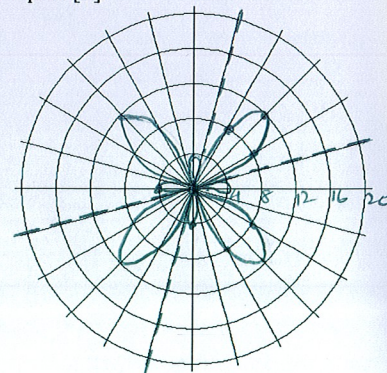
$0 = 4 - 8\cos 4\theta$
$8\cos 4\theta = 4$
$\cos 4\theta = \frac{1}{2}$ ($\frac{\pi}{2}$)

$0 \le \theta \le \frac{\pi}{2} \longrightarrow 0 \le 4\theta \le 2\pi$
$4\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ ①
$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$ ①

[d] Find the value of r (rounded to 1 decimal place) for all common angles in the minimum interval in part [b].
NOTE: You do NOT need to show work, only answers.

θ	r
0	-4
$\frac{\pi}{6}$	8
$\frac{\pi}{4}$	12
$\frac{\pi}{3}$	8
$\frac{\pi}{2}$	-4

($\frac{1}{2}$) POINT EACH

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[d] Sketch the graph on the grid provided below. **You must provide a scale for the polar axis & plot all points from part [c] above.**